C & K 4

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Group Bram

1)

a. $e^x : \mathbb{R} \to \mathbb{R}$, $x+1 \neq 0$, $\mathbb{R} \setminus \{-1\}$

b. No roots, numerator (e^x) kan niet gelijk zijn aan 0 want ln 0 ~~is niet~~ bestaat niet.

c.

$$\lim_{x\to\infty} \frac{e^x}{x+1}. \quad \lim_{x\to\infty} e^x = \infty = \lim_{x\to\infty} x+1,$$

beide differentieerbaar dus $\lim_{x\to\infty} \frac{e^x}{x+1} = \lim_{x\to\infty} \frac{(e^x)'}{(x+1)'}$

$$= \lim_{x\to\infty} \frac{e^x}{1} = \infty.$$

$$\lim_{x\to-\infty} \frac{e^x}{x+1} = \frac{0}{\lim_{x\to-\infty} x+1} = 0.$$

$$\lim_{x\to-1^+} \frac{e^x}{x+1} = e^{-1} \cdot \infty = \infty$$

$$\lim_{x\to-1^-} \frac{e^x}{x+1} = e^{-1} \cdot -\infty = -\infty.$$

d. $$f'(x) = \frac{(x+1)\cdot e^x - e^x}{(x+1)^2} = \frac{x \cdot e^x}{(x+1)^2}.$$

$$f''(x) = \frac{(x+1)^2 \cdot (e^x + xe^x) - (xe^x)(2x+2)}{(x+1)^4}$$

$$= \frac{\cancel{(x+1)^3 \cdot e^x - xe^x}}{\cancel{(x+1)}^4}$$

e. $f'(x) = 0$, dan $x \cdot e^x = 0$, dus $x = 0$ (klopt met $(x+1)^2 \neq 0$).

~~$f''(x) = 0$, dan $(x+1)^3 = 0$ dus $x = -1$ of~~

$\cancel{(x+1)}^3 \cdot e^x = x \cdot e^x$ $\cancel{(x^2+2x+1)(x+1) = x^3+3x^2+3x+1} = x$

$\cancel{(x+1)^3 = x}$ $\cancel{x^3+3x^2+2x+1} = 0.$

$(x^2+1)(x+1) = x^3+x^2+x+1$

(1) (d) $f''(x) \dots = \left((x^2+2x+1)\cdot e^x + (x^2-1)\cdot x \cdot e^x \right) \div (x+1)^4$

$= \left((x^3+x^2+x+1)\cdot e^x \right) \div (x+1)^4$

$= \dfrac{(x^2+1)(x+1)}{(x+1)^4} = \dfrac{x^2+1}{(x+1)^3}$

e. $f''(x) = 0$, dan $x^2+1=0$, $x^2=-1$, kan niet.

f. We found only $x=0$ s.t. $f'(x)=0$

$f(0) = e^0/(0+1) = e^0 = 1$.

g. $f''(0) = \dfrac{0^2+1}{(0+1)^3} = 1 > 0$, so $x=0$ is a local minimum

h. $f''(x) > 0$ geeft $\dfrac{x^2+1}{(x+1)^3} > 0$

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4 b) ii $\dfrac{\delta}{\delta x}\left(\dfrac{\delta}{\delta y} f(x,y)\right) = \dfrac{\delta}{\delta x} \dfrac{x\cdot e^{x/y}}{y^2} = \dfrac{e^{x/y} + x\cdot\dfrac{e^{x/y}}{y}}{y^2}$

$= \dfrac{\dfrac{y\cdot e^{x/y} + x\cdot e^{x/y}}{y}}{y^2}$

$= \dfrac{\dfrac{x\cdot e^{x/y} - y\cdot e^{x/y}}{y}}{y^2}$

$= \dfrac{y\cdot\dfrac{x\cdot e^{x/y}}{y^2} - e^{x/y}}{y^2}$

$= \dfrac{\delta}{\delta y}\left(\dfrac{e^{x/y}}{y}\right) = \dfrac{\delta}{\delta y}\left(\dfrac{\delta}{\delta x} f(x,y)\right)$. $\square$

3 a) e^x is $\mathbb{R} \to \mathbb{R}$, $\sin(x)$ is $\mathbb{R} \to [-1, 1]$

$\cdot$ from $\mathbb{R} \times [-1, 1]$ goes to $\underline{\underline{\mathbb{R}}}$.

b) $e^x \cdot \sin(x) = 0$

$e^x = 0$ or $\sin x = 0$

not possible! $x = k \cdot \pi$ with $k \in \mathbb{Z}$

$\ln(0)$ ↑

So the roots are described by $\{(k \cdot \pi, 0) \mid k \in \mathbb{Z}\}$

c) Using the product rule $f'(x) = e^x \cdot (\sin x)' + (e^x)' \cdot \sin x$

$= e^x \cdot \cos x + e^x \cdot \sin x$

$= e^x (\cos x + \sin x)$

Again using the product rule

$f''(x) = e^x (\cos x + \sin x)' + (e^x)' (\cos x + \sin x)$

$= e^x (\cos x - \sin x) + e^x (\cos x + \sin x)$

$= 2e^x \cdot \cos x.$

d) $f'(x) = 0$ gives $e^x \cdot (\cos x + \sin x) = 0$

$e^x = 0$ or $\cos x = -\sin x$

not possible; $\ln(0)$ ↑ $\cos x = \cos(x + 1\tfrac{1}{2}\pi)$

$f''(x) = 0$ gives $2e^x \cdot \cos x = 0$. Again, $e^x = 0$ is not possible, so we have $\cos x = 0$, then $x \in \{(k + \tfrac{1}{2})\pi \mid k \in \mathbb{Z}\}$.

4. a) i $\quad f(x,y) = \cos(4y - xy)$

$$\frac{\partial}{\delta x} f(x,y) = -\sin(4y-xy)\cdot(4y-xy)'$$
$$= -\sin(4y-xy)\cdot -y$$
$$= y\cdot\sin(4y-xy).$$

$$\frac{\delta}{\delta y}\{f(x,y)) = -\sin(4y-xy)\cdot(4-x)$$
$$= (x-4)\cdot\sin(4y-xy).$$

ii $\quad f(x,y) = e^{x/y}$

$$\frac{\delta f(x,y)}{\delta x} = e^{x/y}\cdot(1/y) = \frac{e^{x/y}}{y}.$$

$$\frac{\delta f(x,y)}{\delta y} = e^{x/y}\cdot -\frac{x}{y^2} = \frac{x\cdot e^{x/y}}{y^2}.$$

$$(x+y)^2\cdot 2x - x^2(2x+y)$$
$$= (x^2+2yx\cancel{x}+y^2)\cdot 2x - 2x^3 - yx^2$$
$$= 2x^3+4yx^2+2y^2x-2x^3-yx^2$$
$$= 3yx^2+2y^2x$$

b) i $\quad \frac{\delta}{\delta x}\left(\frac{\delta}{\delta y} f(x,y)\right) = \frac{\delta}{\delta x}(x-4)\cdot\sin(4y-xy)$

$$= \sin(4y-xy) + (x-4)(\cos(4y-xy)\cdot -y)$$
$$= \sin(4y-xy) + y(4-x)\cdot\cos(4y-xy).$$

$$= \sin(4y-xy) + (4y-xy)\cdot\cos(4y-xy)$$
$$= \sin(4y-xy) + y\cdot(\cos(4y-xy)\cdot(4-x))$$
$$= \frac{\delta}{\delta y}\left(y\cdot\sin(4y-xy)\right) = \frac{\delta}{\delta y}\left(\frac{\delta}{\delta x} f(x,y)\right).$$ $\boxed{I}$

NB: 4b) ii is on page 2.

(5) $$\frac{\delta}{\delta x} f(x,y) = \frac{(x+y)\cdot y - xy}{(x+y)^2} = \frac{y^2}{(x+y)^2}.$$

$$\frac{\delta}{\delta x}\left(\frac{\delta}{\delta x} f(x,y)\right) = \frac{(x+y)^2 \cdot 0 - y^2 \cdot (2x+2y)}{(x+y)^4}$$

$$= -\frac{2y^3 + 2xy^2}{(x+y)^4}.$$

$$\frac{\delta}{\delta y}(f(x,y)) = \frac{x^2}{(x+y)^2}.$$

$$\frac{\delta}{\delta x}\left(\frac{\delta}{\delta y} f(x,y)\right) = \frac{(x+y)^2 \cdot 2x - x^2(2x+2y)}{(x+y)^4}$$

$$= \frac{2x(2yx+y^2) - 2yx^2}{(x+y)^4}$$

$$= \frac{2xy^2 + 2yx^2}{(x+y)^4} = \frac{2xy(y+x)}{(x+y)^4} = \frac{2xy}{(x+y)^3}$$

$$\frac{\delta}{\delta y}\left(\frac{\delta}{\delta y} f(x,y)\right) = -\frac{2x^3 + 2yx^2}{(x+y)^4},$$

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$$x^2 \cdot \left(-\frac{2y^3 + 2xy^2}{(x+y)^4}\right) + 2xy\left(\frac{2xy}{(x+y)^3}\right) + y^2\left(-\frac{2x^3 + 2yx^2}{(x+y)^4}\right)$$

$$= \frac{-2x^2y^3 - 2x^3y^2 + 4x^2y^2(x+y) - 2y^2x^3 - 2y^3x^2}{(x+y)^4}$$

$$= (4x^2y^3 + 4x^3y^2 - 4x^2y^3 - 4x^3y^2) \div (x+y)^4$$

$$= \cancel{(x^2y^3 - x^3y^2) \div (x+y)^4.} \quad 0/4 = 0. \quad \square$$

6\. a) Let $f(x) = x^3 + x + 1$

Then $F(x) = \frac{1}{4}x^4 + \frac{1}{2}x^2 + x$

since $\left(\frac{1}{4}x^4 + \frac{1}{2}x^2 + x\right)' = x^3 + x + 1 = f(x)$.

Then $\int_{-1}^{1} f(x)\, dx = F(1) - F(-1)$

$= \left(\frac{1}{4}1^4 + \frac{1}{2}1^2 + 1\right) - \left(\frac{1}{4}(-1)^4 + \frac{1}{2}(-1)^2 - 1\right)$

$= 1\,{}^3\!/_4 + {}^1\!/_4 = 2.$

b) Let $f(x) = 3\sqrt{x} + \frac{3}{x^2}$

Then $F(x) = 2x^{1\frac{1}{2}} \cancel{+} - \frac{3}{x} = 2x^{1\frac{1}{2}} - 3 \cdot x^{-1}$

since $\left(2x^{1\frac{1}{2}} - 3x^{-1}\right)' = 3\sqrt{x} + {}^3\!/_{x^2} = f(x)$.

Then $\int_{1}^{2} f(x)\, dx = F(2) - F(1)$

$= \left(2 \cdot 2^{1\frac{1}{2}} - {}^3\!/_2\right) - \left(2 \cdot 1^{1\frac{1}{2}} - {}^1\!/_2\right)$

$= 4\sqrt{2} - 3.$

c) Let $f(x) = \sin x + \cos x$.

Then $F(x) = -\cos x + \sin x$, since $(-\cos x + \sin x)' = f(x)$.

Then $\int_{0}^{\pi} f(x)\, dx = F(\pi) - F(0) = (-\cos \pi + \sin \pi) - (-\cos 0 + \sin 0)$

$= (1 + 0) - (-1 + 0) = 2.$

d) Let $f(x) = \frac{-5}{\sqrt{1-x^2}}$. Then $F(x) =$ ~~arcsin~~ $5 \cdot \arcsin x$

since $(5 \cdot \arcsin x)' = \frac{5}{\sqrt{1-x^2}} = f(x)$.

Then $\int_{-1}^{1} f(x)\, dx = F(1) - F(-1)$

$= 5 \cdot \arcsin 1 - 5 \cdot \arcsin -1$

$= 5 \cdot \left(\frac{1}{2}\pi - -\frac{1}{2}\pi\right) = 5\pi.$

7 . a) ~~Let $u = x^n$. Then $du = n x^{n-1} dx$.~~
~~Then $dx = du / n x^{n-1}$~~

~~So $\int_1^\infty \frac{1}{x^n} dx = \int_1^\infty \frac{1}{u} \frac{du}{n x^{n-1}}$ $\quad x^{-n} \quad \frac{1}{-n+1} x^{-n+1}$~~

~~$= \int_1^\infty \frac{\ln|u|}{n x^{n-1}} = \int_1^\infty \frac{\ln|x^n|}{n x^{n-1}}$~~

$$\int_1^\infty \frac{1}{x^n} dx = \lim_{b \to \infty} \int_1^b \frac{1}{x^n} dx = \lim_{b \to \infty} \left. \frac{1}{(-n+1) x^{n-1}} \right]_1^b$$

Since $n \geq 2$ we have $-n+1 < 0$ and $x^{n-1} > 0$.

~~We know lim~~

$$= \lim_{b \to \infty} \left(\underbrace{\frac{1}{(-n+1) b^{n-1}}}_{\text{goes to } 0} - \underbrace{\frac{1}{(-n+1) \cdot 1}}_{\text{goes to } \frac{1}{-n+1}} \right) = \frac{1}{n-1}.$$

b) Let $f(x) = \frac{x \cdot \cos x - \sin x}{x^2}$. Then $F(x) = \frac{\sin x}{x}$.

$$\int_{-\infty}^{-\pi/2} f(x)\, dx = \lim_{t \to -\infty} \int_t^{-\pi/2} f(x)\, dx = \lim_{t \to -\infty} \left. F(x) \right]_t^{-\pi/2}$$

$$= F(-\pi/2) - \lim_{t \to -\infty} F(t) = 2/\pi - 0 = 2/\pi.$$

~~But $\lim_{t \to -\infty} F(t) = \lim_{t \to -\infty} \frac{\sin t}{t}$ does not~~

8. b) $f(x) = 2\sin x \cos x = \sin(2x)$

$F(x) = -\frac{1}{2}\cos(2x)$

since $F'(x) = \left(-\frac{1}{2}\cos(2x)\right)'$

$= -\frac{1}{2} \cdot -\sin(2x) \cdot (2x)'$

$= \sin(2x) = f(x).$

c

NB: yes, there is no page 8!